# Efficient Bayesian Detection of Faint Curved Edges in Noisy Images

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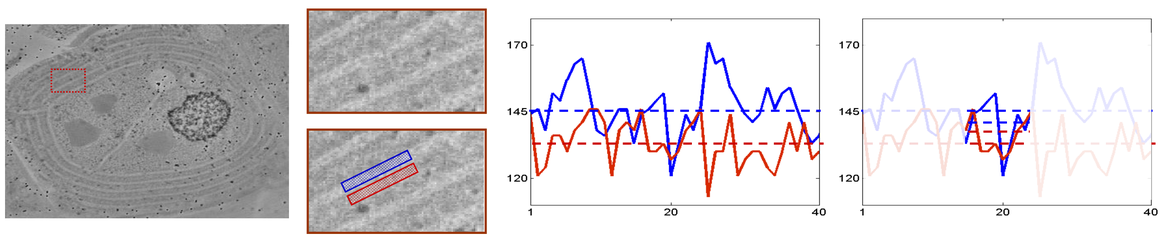
## Goals

**Answer the following fundamental questions:**

- **How faint an edge can be and still be detected?**
- **What is the computational complexity needed for edge detection?**

Faint edges can be detected by matched filters

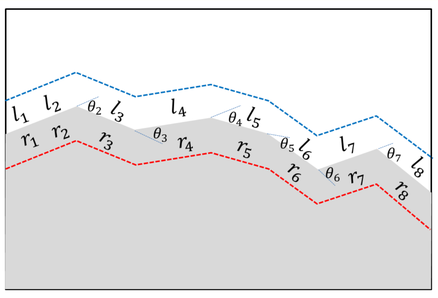



**Differences of oriented means (Galun et al.)**

At all lengths and orientations

## Bayesian Detection

**Matched Filter**



$L$ = length

$w$ = width

$K_L$ = search space size

$N$ = pixels

$\bar{C}$ = Mean Contrast $\theta$ = Shape

$$\frac{P(edge|\bar{C},\theta)}{P(noise|\bar{C},\theta)} > 1 \leftrightarrow \frac{P(\bar{C},\theta|edge)}{P(\bar{C},\theta|noise)} > \sqrt{K_L}$$

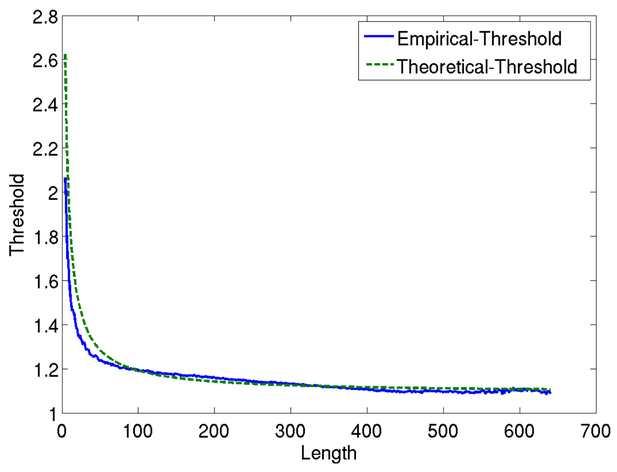
## Minimal Detectable Contrast

**Our Search Space** $K_L \approx 6N \cdot 2^{0.66L}$

**Contrast Threshold:** $\frac{\bar{c}}{\sigma_n} > \sqrt{\frac{2\ln(K_L)}{wL}} = T$

$$\lim_{w \to \infty} T = 0$$

$$\lim_{L \to \infty} T = O(w^{-0.5})$$

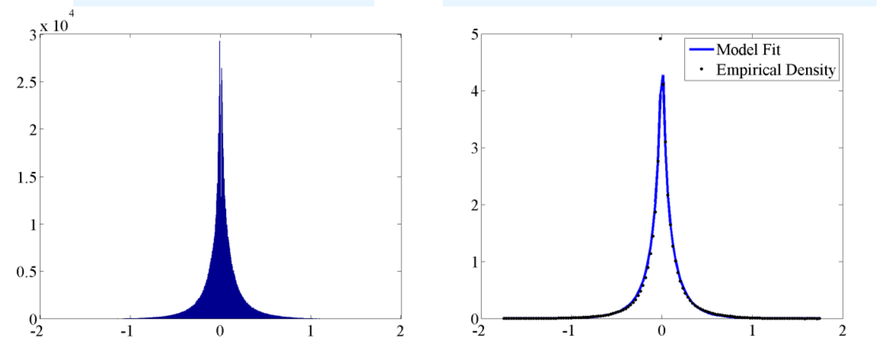



## Contrasts in Natural Images

**Histogram** **Density Function**

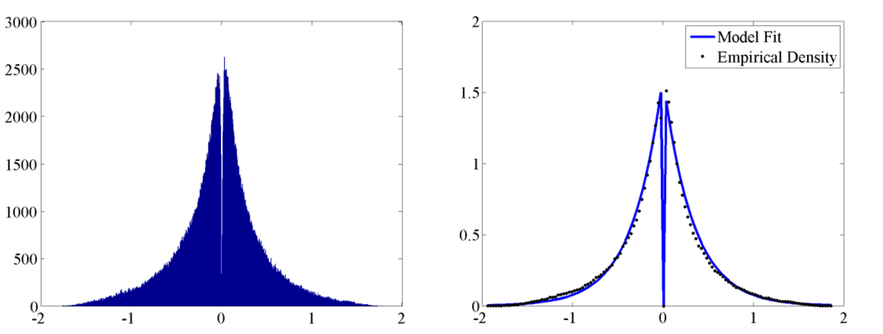
$P(c|noise)$

$$\frac{e^{-\left|\frac{c}{s}\right|^p}}{Z(s,p)}$$




$P(c|edge)$

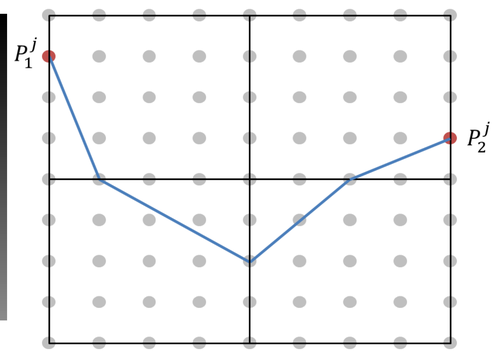
$$1_{c>\alpha} \cdot \frac{e^{-\left|\frac{c}{s}\right|^p}}{Z'(s,p)}$$




## Efficient Detection

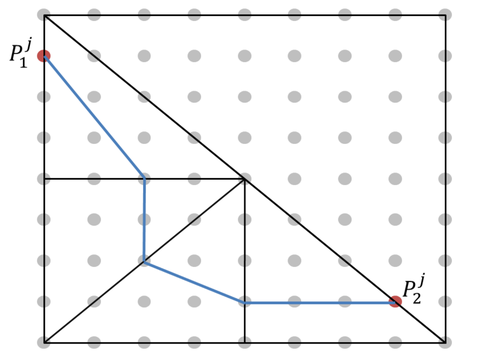
### Quad Pyramid (Alpert at el.)

- **Bottom Level - (5,5) squares**
- **Curve of Level J - Stitching of up to 4 curves of level J-1**
- **Search Space(L)** $\approx 6N \cdot 2^{0.75L}$
- **Levels -** $log_4(N)$




### Triangle Partition Tree (Ours)

- **Bottom Level - (5,5,5) triangles**
- **Curve of Level J - Stitching of up to 2 curves of level J-1**
- **Search Space(L)** $\approx 6N \cdot 2^{0.66L}$
- **Levels -** $log_2(N)$




### Edge Detection Complexity

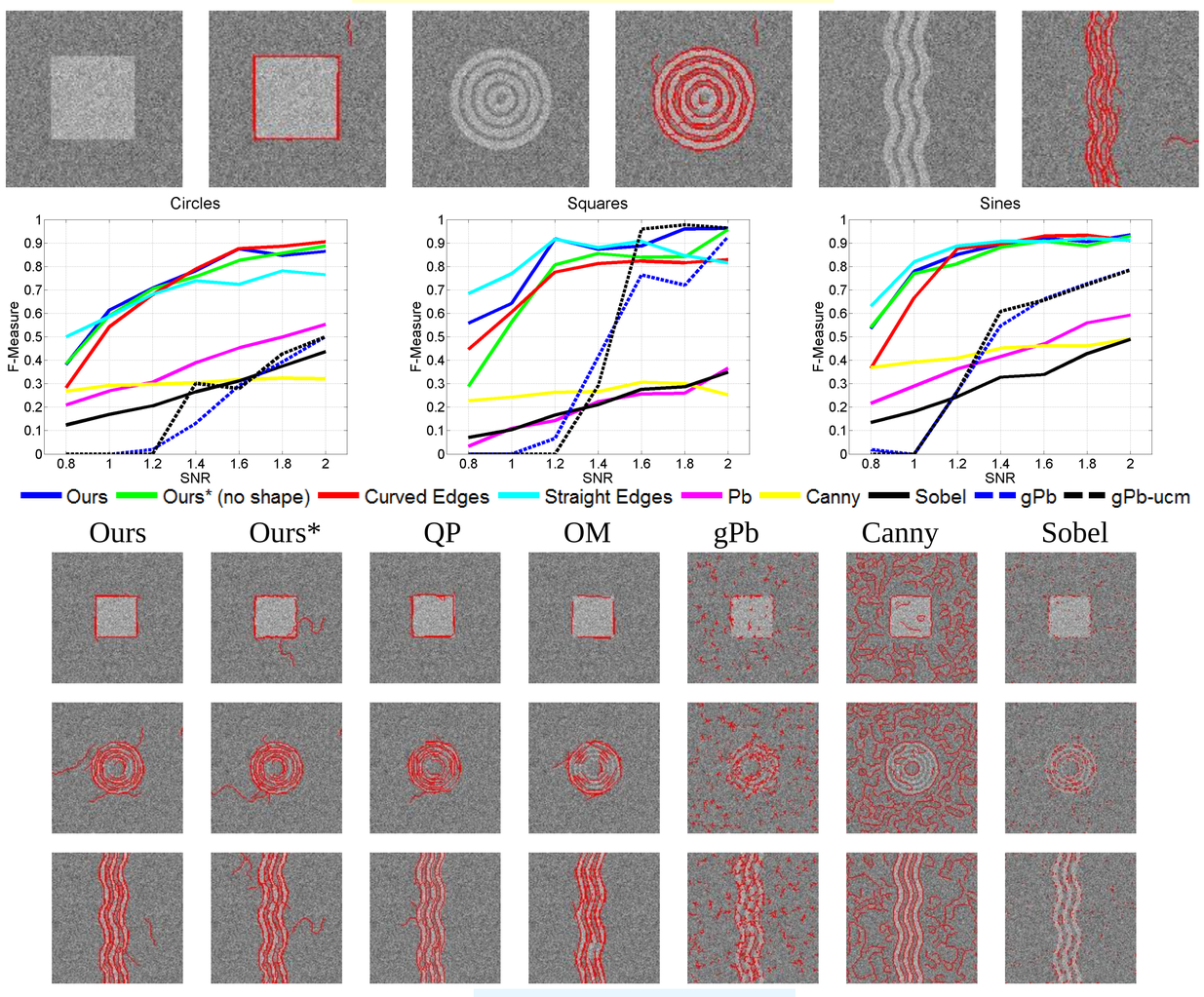
| Algorithm | Complexity |
|---|---|
| Canny | $O(N)$ |
| Oriented Means (OM) | $O(NlogN)$ |
| TPT (Ours) | $O(N^{1.5})$ |
| QP Monotone Curves | $O(N^2)$ |
| QP General Curves | $O(N^{2.5})$ |
| Naive Approach | $O(\exp(N))$ |

$O(N)$

**Quality-Complexity Tradeoff**

$O(exp(N))$

## Results




**Real Images**